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Introduction

Great economic development after the second world war has released a need of development of mathematical methods to support optimization of economic and business processes. Material flow, production, inventory are aspects of a business, which to make it profitable, need to be optimized. Therefore many models of supply chain were created in the mid of 20th century. To mention the most noticeable, we owe to list works of Wagner and Within [36], Brown [5] and Holt, Modigliani, Muth, Simon (HMMS model) [14]. They have laid the foundations for supply chain modelling. Those models, although relatively simple, have become an inspiration for contemporary researcher: to redefine models in order to fit to the current challenges and to analyse them using available computation power, e.g. [28], [18], [23].

Ma and Feng in [23] proposed the dynamical model of demand and inventory with mechanism of demand stimulation and inventory limitation. Nowakowska in [26] showed application of such a model to electrical energy market, Hachuła and Schmeidel in [12] proposed adjustment of a model to the real business case of a product in a decline phase of product life cycle and analysed the stability of equilibrium points in in [9], [11] and [25]. Further analysis of the model on the ground bifurcation theory is continued in [10].

We analyse properties of a given model (1) on the ground of theory of difference equations. Three-dimensional difference systems were studied by many authors, for example in [24], [31], [32], [33]. Under some assumptions such systems can be rewritten as third order difference equations. Asymptotic properties of this type of equations were investigated in [3], [4], [6], [21], [27], [30]. For background of difference equations theory see monographs [2], [1], [7], [16] and [20].

Besides the Introduction, the monograph consists of four chapters and is organized as follows:

In Chapter 1 we introduce definitions and theorems that are used throughout the monograph. We start with a definition of an equilibrium point of the system of difference equations and definitions of different types of stability of the considered system. Moreover, we recall standard sufficient conditions guaranteeing stability of the equilibrium point in case of a continuous or smooth function defining the system of difference equations. The last part of Chapter 1 dealt with one-parameter bifurcation of the system. We recall only two types one-parameter bifurcation because only these bifurcations occurred in our system. We present the standard sufficient conditions of existence of transcritical and Neimark-Sacker bifurcations.

In Chapter 2, we consider the system of difference equations given by formula

$$\begin{cases} x_{n+1} &= \left[\frac{AT}{(A+1)T-y_n}\right]^k x_n \\ y_{n+1} &= y_n - x_n + z_n \\ z_{n+1} &= \alpha x_n + (1-\alpha) z_n \end{cases}, \tag{1}$$

where x_n, y_n, z_n are variables and $A, T, k > 0$ and $\alpha \in (0,1)$ are parameters. Formulation of system (1) and hence restrictions for variables and parameters come from its application to social science - microeconomics and management engineering, which are presented in details in Chapter 4. Significant feature of system (1) is possession of only nonisolated equilibrium points. Hence, the equilibrium points are nonhyperbolic and in consequence Grobman-Hartman theorem cannot be applied to study their stability. It turned out to be necessary to apply other techniques like Lyapunov theory and centre manifold theory. System (1) has invariant plane, what let to reduce three dimensional system to a planar one. A theorem on dependencies between stability of equilibrium points of system (1) and the planar system related to it is formulated and proved, as well as theorem on stability and instability of the equilibrium points.

In Chapter 3, we analyse one-parameter bifurcation of the planar system related to (1). Conditions which imply an occurrence of bifurcation are provided. Occurrence of bifurcation of Neimark-Sacker is proven. The proof is supported with numerical analysis, including bifurcation diagrams and phase portraits. Moreover, the existence of transcritical bifurcation is discussed.

In Chapter 4, system (1) is considered from perspective of their applications with focus on microeconomic phenomenon - demand-inventory management related to the product life cycle. Microeconomic background is provided. Presentation of potential applications are augmented with simulations and examples.

Numerical analysis collects technical aspect of the research presented in previous chapters. We provide programs and functions used for plotting graphs and preparing simulations. Numerical analysis was performed using Matcontm, Matlab R2016a, Maxima 5.38.0 (according to [8], [22] and [35] respectively) and MS Excel 2010. All figures and tables are own elaborations, unless otherwise stated.

It is our pleasure to thank Professor Marek Galewski for encouraging us to write this monograph. We would like to thank our families for their continuous support.